

$$B = \frac{1}{\beta} \operatorname{imag} \left[\lim_{s \rightarrow \alpha + i\beta} \left[(s-\alpha)^2 + \beta^2 \right] \frac{N(s)}{D(s)} \right]$$

$$s = \alpha + i\beta$$

$$s = -\xi \omega_n + i \omega_d$$

$$B = \frac{1}{\omega_d} \operatorname{imag} \left[\lim_{s \rightarrow \alpha + i\beta} \frac{\omega_n^2 P}{s^2 + P s} \right] \rightarrow \frac{1}{\omega_d} \operatorname{ima} \left[\frac{\omega_n^2 P}{(-\xi \omega_n + i \omega_d)^2 - \xi \omega_n P + j \omega_d P} \right]$$

$$B = \frac{1}{\omega_d} \operatorname{imag} \left[\frac{\omega_n^2 P}{\xi^2 \omega_n^2 - \omega_d^2 - j 2 \xi \omega_n \omega_d - \xi \omega_n P + j \omega_d P} \right]$$

$$B = \frac{1}{\omega_d} \operatorname{imag} \left[\frac{\omega_n^2 P}{\xi^2 \omega_n^2 - \omega_d^2 - \xi \omega_n P + j (\omega_d P - 2 \xi \omega_n \omega_d)} \right] \quad \begin{array}{l} \text{Multiplica por el} \\ \text{conjugado} \end{array}$$

$$B = \frac{1}{\omega_d} \operatorname{imag} \left[\frac{P \omega_n^2 (\xi^2 \omega_n^2 - \xi \omega_n P - \omega_d^2) + j P \omega_d \omega_n (2 \xi \omega_n - P)}{\xi^4 \omega_n^4 - 2 \xi^3 \omega_n^3 P + \xi^2 (P^2 + 2 \omega_d^2) \omega_n^2 - 2 \xi P \omega_d^2 \omega_n + (P^2 + \omega_d^2) \omega_d^2} \right]$$

Extrayendo la Componente Imaginaria

$$B = \frac{P \omega_n^2 (2 \xi \omega_n - P)}{\xi^4 \omega_n^4 - 2 \xi^3 \omega_n^3 P + \xi^2 (P^2 + 2 \omega_d^2) \omega_n^2 - 2 \xi P \omega_d^2 \omega_n + (P^2 + \omega_d^2) \omega_d^2}$$

$$\hookrightarrow \omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$B = \frac{P^2 - 2 \xi \omega_n P}{2 \xi \omega_n P - P^2 - \omega_n^2}$$

$$\frac{B\alpha + C}{\beta} = \frac{1}{\beta} \operatorname{real} \left[\lim_{s \rightarrow \alpha + i\beta} \left[(s-\alpha)^2 + \beta^2 \right] \frac{N(s)}{D(s)} \right]$$

Extrayendo Parte Real

$$\frac{B\alpha + C}{\beta} = \frac{1}{\omega_d} \left[\frac{P \omega_n^2 (-\xi^2 \omega_n^2 - \xi \omega_n P - \omega_d^2)}{\xi^4 \omega_n^4 - 2 \xi^3 \omega_n^3 P + \xi^2 (P^2 + 2 \omega_d^2) \omega_n^2 - 2 \xi P \omega_d^2 \omega_n + (P^2 + \omega_d^2) \omega_d^2} \right]$$

$$\frac{B\alpha + C}{\beta} = \frac{-(2 \xi^2 \omega_n - \xi P - \omega_n) P \omega_n}{(2 \xi P \omega_n - P^2 - \omega_n^2) \sqrt{1 - \xi^2} \omega_n}$$

$$A = \lim_{s \rightarrow 0} \frac{\omega_n^2 P}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(s + P)} = 1$$

$$D = \lim_{s \rightarrow -P} \frac{\omega_n^2 P}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$D = \frac{\omega_n^2 P}{-P(P^2 - 2\zeta\omega_n P + \omega_n^2)} =$$

$$D = \frac{-\omega_n^2}{P^2 - 2\zeta\omega_n P + \omega_n^2}$$

Respuesta Transitoria (temporal)

$$y(t) = A + B e^{-\zeta\omega_n t} \cos(\omega_d t) + \frac{B\alpha + C}{\beta} e^{-\zeta\omega_n t} \sin(\omega_d t) + D e^{-pt}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$